ELSEVIER
CrossMark
Available online at www.sciencedirect.com
ScienceDirect
Procedia Materials Science 3 (2014) 1191 – 1196
Procedia Materials Science
www.elsevier.com/locate/procedia

20th European Conference on Fracture (ECF20)

BEM implementation of a quasistatic and rate-independent non-associative model of mixed-mode interface-crack growth

Vladislav Mantič*, Christos G. Panagiotopoulos

Grupo de Elasticidad y Resistencia de Materiales, Escuela Técnica Superior de Ingeniería
Universidad de Sevilla, Camino de los Descubrimientos s/n, 41092 Sevilla, Spain

Abstract

The present authors and coworkers have recently developed a new physically and mathematically well justified and efficient approach for interface crack onset and propagation, implemented in finite and boundary element method (FEM and BEM) codes and applied to several problems of engineering interest. This approach borrows concepts from damage mechanics, such as the damage variable, from plasticity, as kinematic hardening, and from interface fracture mechanics, as fracture energy dependent on the fracture mode mixity. The computational implementation is based on recursive minimizations of a total energy functional, which can be computed by FEM and BEM. Global or specific local minimizations lead to different solution types, the energetic and stress driven solutions, respectively. In opposite to the associative models, where interface plasticity is explicitly taken into account by a plastic slip variable, applied to mixed-mode crack propagation problems by the present authors so far, it seems that non-associative models have the advantage of ending up at easier (e.g. smooth instead of non-smooth) and reduced (e.g. elimination of plasticity variable) minimization problems. An implementation of such a non-associative model in a collocation BEM code is presented and applied to an engineering problem of delamination.

Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering

Keywords: adhesive contact, interface damage, rate-independent, non-associative model, boundary element method, quadratic programming

1. Introduction

Interfaces are present in many engineering materials, as composites, and structures, as multilayers and bonded structures. An interface represents frequently a weak surface where a crack, sometimes also referred to as debond or delamination, can appear. Such a crack is frequently trapped at the interface and propagates along it in mixed mode due to an asymmetry caused by dissimilar materials on both sides of the interface, relative orientation of the interface with respect to the global geometry and load configuration. These facts make the interface crack propagation very different from that of classical cracks growing in homogenous materials usually in opening mode I. In several well-known and carefully carried out experiments by Banks-Sills and Ashkenazi (2000); Evans et al. (1990); Liechti and

* Corresponding author. Tel.: +34-95-448-2135 ; fax: +34-95-446-1637.
E-mail address: mantic@us.es

2211-8128
Selection and peer-review under responsibility of the Norwegian University of Science and Technology (NTNU), Department of Structural Engineering
doi:10.1016/j.mspro.2014.06.194

Chai (1992), see also Mantič (2008), it has been shown that the fracture energy (fracture toughness) G_c of interface cracks depends strongly on the fracture mode mixity. This fracture mode mixity is typically measured by an angle, given either by the angle of the interface traction vector ahead of the crack tip (ψ_σ), or by the ratio of mode I and II contributions to the energy release rate (ψ_G), see Mantič and París (2004); Távara et al. (2011); Mantič et al. (2014). An increase of G_c in mixed mode in comparison with pure mode I might be explained by some inelastic processes at the interface and its neighborhood, a main contribution to this increase being usually attributed to a larger plastic zone in mode II due to interface shear, cf. Evans et al. (1990); Liechti and Chai (1992); Tveergard and Hutchinson (1993).

In the present work the interface is modeled by a very thin adhesive layer, which under a loading can be partially or fully damaged in some parts. An interface crack can be represented by a fully damaged part of the layer. The evolution of damage in the adhesive layer is modeled as a quasistatic (neglecting inertial effects) and rate-independent process. The description of the layer damage is based on a scalar damage quantity z defined along interfaces, which takes values from the interval [0, 1], with $z = 0$ meaning no adhesion due to the total damage of the adhesive and $z = 1$ meaning no damage. During a damage evolution the damage variable decays in time, i.e. $\dot{z} < 0$, and it is assumed that during decrease from $z = 1$ to $z = 0$ the amount of energy per unit area equal to G_c has to be released and is dissipated at the interface.

Recently, the present authors and co-workers have developed a new approach to model damage evolution at interfaces under mixed mode. This approach is based on the minimization of the sum of the stored strain energy in the bulk and interface (represented by a thin adhesive layer) and the energy dissipated at the interface and, in the case of viscoelastic materials, also in the bulk, see Panagiotopoulos et al. (2013); Roubíček et al. (2013a,b, 2014a,b); Vodička and Mantič (2011); Vodička et al. (2014). While we have initially carried out an thorough research on interface damage evolution under mixed mode by employing associative models, where interface plasticity has explicitly been taken into account by means of an additional variable defining interface plastic slip, it seems that non-associative models, proposed in Roubíček et al. (2014a) and further developed by Kružík et al. (2014), are able to model such interface damage evolution as well, with the advantage of ending up at easier (e.g. smooth instead of non-smooth) and reduced (e.g. elimination of plasticity slip variable) minimization problems. Thus, the goal of the present work is to carry out a further numerical study of such non-associative models. Section 2 briefly presents a non-associative model for interface damage evolution under mixed mode, different functional dependencies of G_c on fracture-mode-mixity angle are described in Section 3. Finally, a numerical implementation of the model and 2D simulations are presented in Section 4.

2. Non-associative model for interface damage evolution under mixed mode

Let Ω be a 2D elastic domain, with a Lipschitz boundary $\partial\Omega = \Gamma_D \cup \Gamma_N \cup \Gamma_C$, bonded, by a linear elastic-brittle adhesive layer, to an outer rigid obstacle along the so-called interface Γ_C. The time dependent Dirichlet and Neumann boundary conditions, $w(t)$ and $f(t)$, respectively, are prescribed on Γ_D and Γ_N for the displacement u and traction vector $T^n(u)$. Let $\kappa_n \geq 0$ and $\kappa_t \geq 0$ be the normal and tangential stiffnesses of the adhesive layer, $[\![u]\!]_n$ and $[\![u]\!]_t$ denote the normal and tangential displacement jumps across the interface Γ_C, and $[\![u]\!]_n \geq 0$ express the classical Signorini non-penetration condition on Γ_C. For the sake of simplicity, body forces are neglected. A generalization to the case of two or more elastic subdomains bonded along their interface Γ_C is straightforward.

We consider an evolution in time t governed by the potential energy functional, including the strain energy stored in the linear elastic domain Ω (represented by a boundary integral due to the Clapeyron theorem) and at the interface Γ_C (represented by a linear elastic-brittle adhesive layer),

$$\mathscr{E}(t,u,z) = \int_{\partial\Omega} \frac{1}{2}\, u \cdot T^n(u)\, \mathrm{d}S - \int_{\Gamma_N} f(t) \cdot u\, \mathrm{d}S + \int_{\Gamma_C} \frac{1}{2} z \left(\kappa_n [u]_n^2 + \kappa_t [u]_t^2 \right) \mathrm{d}S, \tag{1}$$

and the rate of dissipation due to interface damage

$$\mathscr{R}(u,\dot{z}) := \int_{\Gamma_C} G_c\big(\psi([u])\big) \big|\dot{z}\big|\, dS. \tag{2}$$

Considering, for the sake of simplicity, a constant time-step $\tau > 0$, the energetic solution can be computed by an implicit time-discretization defining the recursive global (non-convex) minimization problem for (u, z)

$$\min_{u|_{\Gamma_D}=w(t)\ \&\ [\![u]\!]_n|_{\Gamma_C}\geq 0\ \&\ 0\leq z\leq z^{k-1}} \mathscr{E}(k\tau, u, z) + \mathscr{R}(u^{k-1}, z-z^{k-1}) \tag{3}$$

to be solved successively for $k = 1, \ldots$, starting from some initial conditions $u^0 = u_0$ and $z^0 = z_0$, and giving the solution (u^k, z^k) at the time level k. This model is non-associative due to the fracture energy $G_c([\![u]\!])$ in (2) dependent on the fracture-mode-mixity angle ψ. This is different from some previous works by the authors and coworkers, Roubíček et al. (2013a); Panagiotopoulos et al. (2013), where an associative fracture-mode-mixity sensitive model was developed and implemented by using a constant G_c and an additional internal variable, namely interface plastic slip, which produces an additional energy dissipation due to (kinematic hardening) plasticity development in shear or mixed mode.

Nevertheless, the energetic solution from (3) requires to solve a quite difficult non-convex minimization problem and, moreover, sometimes predicts interface damage for unrealistically too small load values. Therefore, in the present work we follow Kružík et al. (2014), where an semi-implicit time-discretization scheme was introduced using a popular fractional-step-like strategy, with alternating convex minimizations in each time level k, first for u and then for z,

$$\min_{u|_{\Gamma_D}=w(t)\ \&\ [\![u]\!]_n|_{\Gamma_C}\geq 0} \mathscr{E}(k\tau, u, z^{k-1}) \tag{4a}$$

and, denoting the unique solution as u^k,

$$\min_{0\leq z\leq z^{k-1}} \mathscr{E}(k\tau, u^k, z) + \mathscr{R}(u^k, z-z^{k-1}), \tag{4b}$$

denoting its (possibly not unique) solution as z^k. Similar alternating convex minimizations was proposed and studied for the above mentioned associative model by Roubíček et al. (2013b, 2014b); Vodička et al. (2014), showing that it provides a new concept of maximally dissipative local solution. Mathematical analysis in Kružík et al. (2014) shows that such a non-associative model may need a certain (even vanishing) amount of viscosity in the bulk, in order to guarantee convergence. On the other hand, as can be seen in Roubíček et al. (2013b), even the simplified inviscid algorithm, with no viscosity considered, showed to be numerically stable and provided good results. Thus, for the sake of simplicity, we adopt here the inviscid version of the non-associative model defined by (1), (2) and (4).

3. Phenomenological laws for interface fracture energy G_c

A key feature of the present model is its fracture-mode-mixity sensitivity, defining G_c as a suitable function of a fracture-mode-mixity angle, e.g., the angle ψ_G, cf. Távara et al. (2011); Mantič et al. (2014),

$$\tan\psi_G = \sqrt{\frac{\kappa_t}{\kappa_n}}\left|\frac{[\![u]\!]_t}{[\![u]\!]_n}\right|, \quad \text{for} \quad [u]_n > 0. \tag{5}$$

In engineering applications the phenomenological law of G_c proposed by Hutchinson and Suo (1992), cf. Banks-Sills and Ashkenazi (2000), is usually applied,

$$G_c(\psi_G) = G_{Ic}(1 + \tan^2((1-\lambda)\psi_G)), \tag{6}$$

where $G_{Ic} = G_c(0°)$ gives the fracture energy in mode I, and λ is the so-called mode sensitivity parameter, $0 \leq \lambda \leq 1$. A moderately strong fracture-mode sensitivity occurs when the ratio G_{IIc}/G_{Ic} is about 5-10 (see Fig. 1(a)), with $G_{IIc} = G_c(90°)$ the fracture energy in mode II, which happens for λ about 0.2-0.3. A numerical implementation of (6) in the above non-associative model was presented in Kružík et al. (2014).

In a theoretical study of the behaviour of the above mentioned kinematic-hardening associative model, including an interface plastic slip variable, the following functional dependence of $G_c(\psi_G)$ was deduced, Vodička and Mantič (2011); Panagiotopoulos et al. (2013); Vodička et al. (2014),

$$G_c(\psi_G) = \begin{cases} G_{Ic}, & \text{for } 0 \leq \psi_G \leq \arcsin\dfrac{\sigma_{t,\text{yield}}}{\sqrt{2\kappa_t G_{Ic}}}, \\ \dfrac{2G_{Ic}(\kappa_t+k_H) - \sigma^2_{t,\text{yield}}}{2(\kappa_t+\kappa_H+\kappa_H\tan^2\psi_G)}\left(1+\tan^2\psi_G\right) & \text{for } \arcsin\dfrac{\sigma_{t,\text{yield}}}{\sqrt{2\kappa_t G_{Ic}}} \leq \psi_G \leq \dfrac{\pi}{2}, \end{cases} \tag{7}$$

where $\sigma_{\mathrm{t,yield}}$ is the tangential yield stress and κ_{H} the plastic modulus of kinematic hardening of the interface. From (7) the maximum value of G_{c} is given by

$$G_{\mathrm{IIc}} = G_{\mathrm{c}}(90^\circ) = G_{\mathrm{Ic}}\left(1 + \frac{\kappa_{\mathrm{t}}}{\kappa_{\mathrm{H}}}\right) - \frac{\sigma^2_{\mathrm{t,yield}}}{2\kappa_{\mathrm{H}}}. \tag{8}$$

In the present non-associative model we will test both functional dependencies defined in (6) and (7), the former governed by one parameter λ and the latter by two parameters $\sigma_{\mathrm{t,yield}}$ and κ_{H}, in addition to the parameters G_{Ic}, κ_n and κ_t of a basic linear elastic-brittle model which is (originally) insensitive to mode mixity. Both plots of the normalized fracture energy $G_{\mathrm{c}}(\psi_{\mathrm{G}})/G_{\mathrm{Ic}}$ in Fig. 1 qualitatively represent the behaviour observed in experiments by Banks-Sills and Ashkenazi (2000); Evans et al. (1990); Hutchinson and Suo (1992); Liechti and Chai (1992).

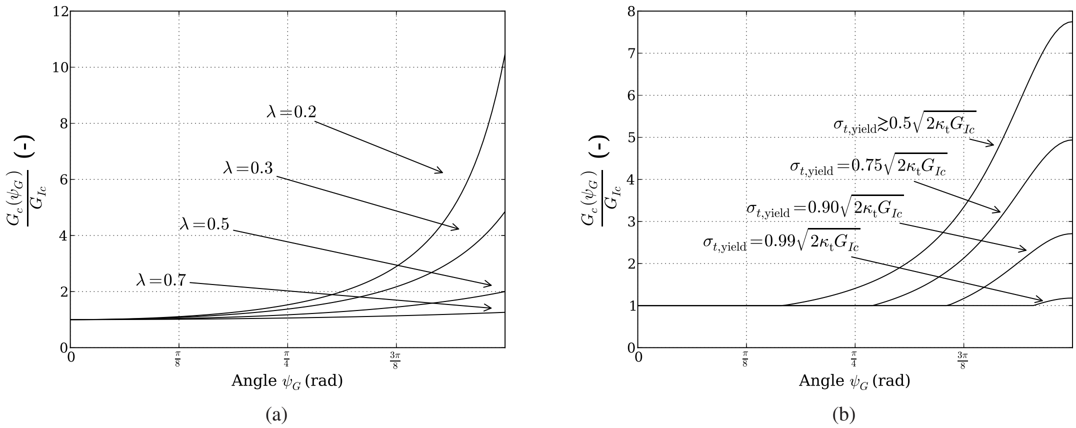

Fig. 1. $G_{\mathrm{c}}(\psi_{\mathrm{G}})/G_{\mathrm{Ic}}$, (a) Influence of λ for the Hutchinson-Suo law in (6), (b) Influence of $\sigma_{\mathrm{t,yield}}$, taking κ_{H}=0.125κ_{t} for the law in (7).

4. Example

A BEM code, Panagiotopoulos (2010), has been further developed by the present authors and coworkers to include different energy based approaches for the prediction of crack onset and propagation at interfaces, Panagiotopoulos et al. (2013); Roubíček et al. (2013a, 2014b); Kružík et al. (2014). The behaviour of the present non-associative model is studied in a relatively simple plane strain example, Fig. 2, motivated by the pull-push shear experimental test used in engineering practice, Cornetti and Carpinteri (2011). The elastic material of the bulk is an aluminium with the elasticity modulus E = 70 GPa and Poisson's ratio ν = 0.35. The basic parameters of the adhesive layer are: κ_{n} =150 GPa/m, $\kappa_{\mathrm{t}} = \kappa_{\mathrm{n}}/2$, G_{Ic} = 187.5 J/m^2.

On the right-hand side of the rectangle, denoted as Γ_{DN}, mixed boundary conditions are prescribed, uniform normal displacements (increasing with time) and zero tangential tractions. All the other boundary parts are traction free, defining the Neumann boundary Γ_{N}, except for the bonded surface Γ_{C} on a portion of the bottom part of the rectangle. The boundary element mesh used to discretize $\partial\Omega$ has 128 elements, 60 and 4 elements along each horizontal and vertical side, respectively. Thus, Γ_{C} is discretized by 54 elements.

The maximum value of fracture energy is G_{IIc}=4.36 G_{Ic}. This correspond to λ=0.318 in the Hutchinson-Suo model (6), Fig. 3(a). $G_{\mathrm{c}}(\psi_{\mathrm{G}})$ defined by (7) is parameterized by a coefficient μ giving $\kappa_{\mathrm{H}}=\frac{\mu}{1-\mu}\kappa_{\mathrm{t}}$. $\sigma_{\mathrm{t,yield}}$ is calculated from (8). The problem is solved for several values of μ shown in Fig. 3(a).

The computed evolution of relevant energies considering $G_{\mathrm{c}}(\psi_{\mathrm{G}})$ defined by (7) for μ=0.1 are shown in Fig. 3(b).

A comparison of the resultant force applied on the right-hand side of the rectangular body Γ_{DN} versus the prescribed displacement therein is shown in Fig. 4(a) for different models considered. Namely, curve [1] stands for (6); curves [2], [3] and [4] for (7) with μ=0.1, 0.07 and 0.15, respectively. Finally, [5] stands for a particular case of the fracture-mode insensitive model with G_{IIc}=G_{Ic}. Very similar result are obtained in the cases [1]-[4], with a slight difference for

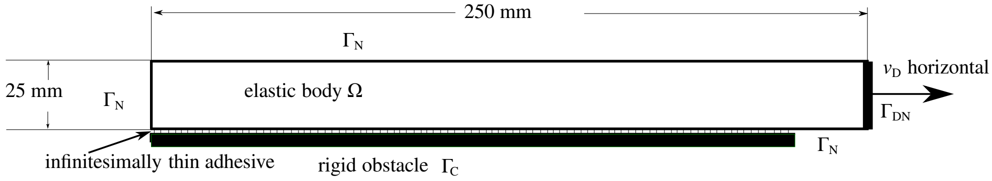

Fig. 2. Geometry and boundary conditions of the 2D problem solved.

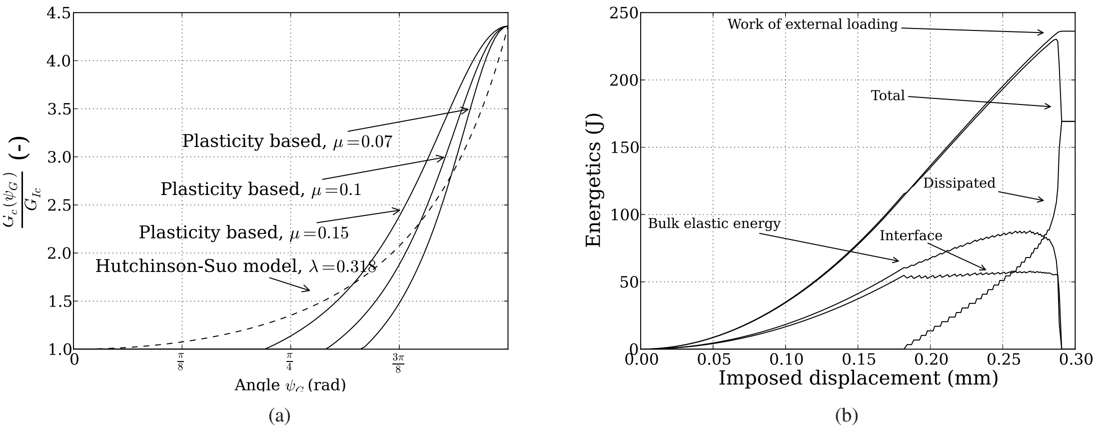

Fig. 3. (a) Normalized fracture energy $G_c(\psi_G)/G_{Ic}$ as a function of ψ_G from (6) and (7), taking λ=0.318 in the former and $\mu = 0.07, 0.1, 0.15$ in the latter. (b) Evolution of relevant energies using (7) with μ=0.1.

the largest values of the prescribed displacements. Fig. 4(b) presents the distribution of the ratio of the actual energy dissipated (per unit area) at an interface point to G_{Ic}. As can be observed, in the cases [1]-[4], the interface breakage occurred essentially in mode II along the approximately two-thirds (placed on the right-hand side) of Γ_c, whereas it occurred in a mixed mode along approximately one-third (placed on the left-hand side) of Γ_c. With reference to the case (5), interface damage initiates for a much smaller value of the resultant force than in the other cases considered and also the displacement leading to the total debond of the rectangular body is much smaller, as could be expected.

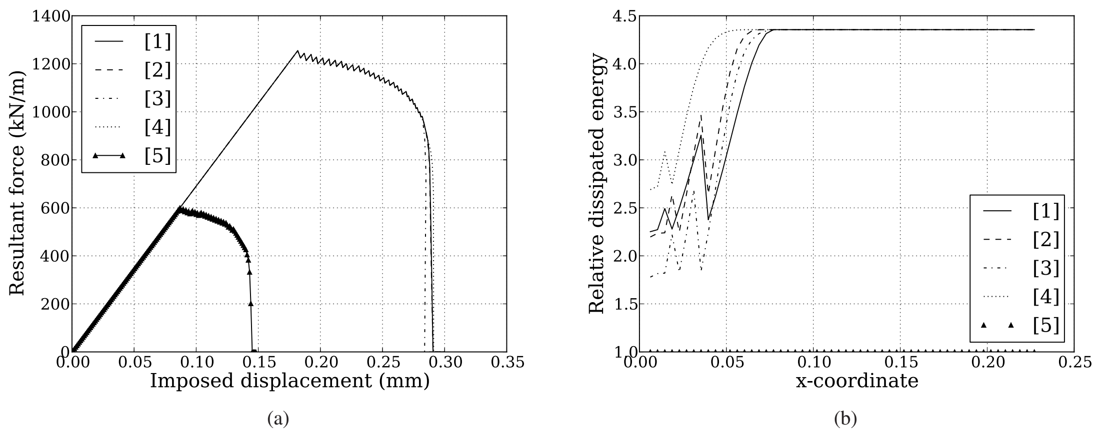

Fig. 4. (a) Evolution of the resultant force with prescribed horizontal displacement on Γ_{DN}. (b) Distribution of ratio G_c/G_{Ic} along Γ_C, $G_c/G_{Ic} \approx 1$ indicates breakage in Mode I, $G_c/G_{Ic} \approx 4.36$ indicates breakage in Mode II.

5. Conclusions

A non-associative model for debonding and delamination problems developed recently by the present authors and coworkers has been studied here. The model has been implemented in a BEM code and tested in a 2D debonding problem. In particular, two different laws for fracture energy as a function of an angle measuring fracture-mode-mixity have been compared. A comparison of the non-associative model using (7) with the associative model, which includes an internal variable of interface plastic slip, would also be interesting and is expected to be presented in a forthcoming work.

Acknowledgements

This contribution has been supported by the Junta de Andalucía and Fondo Social Europeo (Proyecto de Excelencia P08-TEP-4051) and the Spanish Ministry of Economy and Competitiveness (Project MAT2012-37387). CGP would like to acknowledge support of Geomec Engineering AS (Stavanger, Norway), where he was hosted as an employee for a couple of months during the work development.

References

Banks-Sills, L., Ashkenazi, D., 2000. A note on fracture criteria for interface fracture. International Journal of Fracture 103, 177–188.

Cornetti, P., Carpinteri, A., 2011. Modelling the FRP-concrete delamination by means of an exponential softening law. Engineering Structures 33, 1988–2001.

Evans, A., M. Rühle, B.D., Charalambides, P., 1990. The fracture energy of bimaterial interfaces. Metallurgical Transactions A 21, 2419–2429.

Hutchinson, J.W., Suo, Z., 1992. Mixed mode cracking in layered materials, in: Hutchinson, J.W., Wu, T. (Eds.), Advances in Applied Mechanics. Academic Press, New York, pp. 63–191.

Kružík, M., Panagiotopoulos, C.G., Roubíček, T., 2014. Quasistatic adhesive contact delaminating in mixed mode and its numerical treatment. Mathematics and Mechanics of Solids (in print), DOI: 10.1177/1081286513507942.

Liechti, K., Chai, Y., 1992. Asymmetric shielding in interfacial fracture under in-plane shear. Journal of Applied Mechanics 59, 295–304.

Mantič, V., 2008. Discussion on the reference length and mode mixity for a bimaterial interface. Journal of Engineering Materials and Technology 130, 045501–1–2.

Mantič, V., París, F., 2004. Relation between SIF and ERR based measures of fracture mode mixity in interface cracks. International Journal of Fracture 130, 557–569.

Mantič, V., Távara, L., Blázquez, A., Graciani, E., París, F., 2014. Application of a linear elastic-brittle interface model to the crack initiation and propagation at fibre-matrix interface under biaxial transverse loads. ArXiv preprint. arXiv:1311.4596 .

Panagiotopoulos, C.G., 2010. Open BEM Project. http://www.openbemproject.org/ .

Panagiotopoulos, C.G., Mantič, V., Roubíček, T., 2013. BEM solution of delamination problems using an interface damage and plasticity model. Computational Mechanics 51, 505–521.

Roubíček, T., Kružík, M., Zeman, J., 2014a. Delamination and adhesive contact models and their mathematical analysis and numerical treatment, in: Mantič, V. (Ed.), Mathematical Methods and Models in Composites (Chap. 9). Imperial College Press, pp. 349–400.

Roubíček, T., Mantič, V., Panagiotopoulos, C.G., 2013a. Quasistatic mixed-mode delamination model. Discrete and Continuous Dynamical Systems - Series S 6, 591–610.

Roubíček, T., Panagiotopoulos, C.G., Mantič, V., 2013b. Quasistatic adhesive contact of visco-elastic bodies and its numerical treatment for very small viscosity. ZAMM - Zeitschrift für Angewandte Mathematik und Mechanik 93, 823–840.

Roubíček, T., Panagiotopoulos, C.G., Mantič, V., 2014b. Local-solution approach to quasistatic rate-independent mixed-mode delamination. Mathematical Models and Methods in Applied Sciences (submitted) .

Távara, L., Mantič, V., Graciani, E., París, F., 2011. BEM analysis of crack onset and propagation along fiber-matrix interface under transverse tension using a linear elastic-brittle interface model. Engineering Analysis with Boundary Elements 35, 207–222.

Tveergard, W., Hutchinson, J., 1993. The influence of plasticity on mixed mode interface toughness. Journal of the Mechanics and Physics of Solids 41, 1119–1135.

Vodička, R., Mantič, V., 2011. An energetic approach to mixed mode delamination problem – an SGBEM implementation, in: Výpočty konstrukcí metodou konečných prvků 2011, Západočeská univerzita v Plzni, Plzeň, ISBN 978-80-261-0059-1.

Vodička, R., Mantič, V., Roubíček, T., 2014. Energetic versus maximally-dissipative local solutions of a quasi-static rate-independent mixed-mode delamination model. Meccanica (submitted) .